A NEW ZONAL METHOD OF ANALYZING AND CALCULATING THE RADIATION OF HEAT

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The principles of a new zonal method are shown by which various characteristics of heat radiation can be determined, in an extension of the problem to systems with an absorbing and dispersing medium.

The zonal methods proposed and developed in [1-8] are now in wide use for determining both local and mean energy characteristics of heat radiation.

Important here is the extraction of optics-geometrical resolvent functions which are the same for optics-geometrically similar systems. This problem has been explored rather thoroughly in the case of gray bodies with a transparent medium, but in the case of gray bodies with an absorbing and dispersing medium the problem is much more complicated [3].

In view of this, the author felt the need to develop a new zonal method of determining both local and mean characteristics of heat radiation. The results of this effect are shown here. A solution has been obtained to the general problem of heat radiation in a system of gray bodies with diffusive surfaces and an isotropically absorbing and dispersing medium, where densities of the intrinsic radiation in one part of the medium and the boundary surface are given while volume and surface densities of the resultant radiation are given in the other part. The following expressions have been obtained for local surface densities of the incident radiation flux $E_i(m)$ at point m on the boundary surface

$$E_i(\mathrm{m}) = \int_{F_0} \tilde{E}_c(c_0)\,\tilde{P}_1(c_0\mathrm{m})\,dF(c_0) + \int_{V_0} \tilde{\eta}_c(\mathrm{h}_0)\,\tilde{P}_2(\mathrm{h}_0\mathrm{m})\,dV(\mathrm{h}_0) \quad (1)^*$$

and for the volume density of the incident radiation flux $\eta_i(b)$

$$\eta_i(b) = \int_{F_0} \tilde{E}_c(c_0)\,\tilde{P}_3(c_0 b)\,dF(c_0) + \int_{V_0} \tilde{\eta}_c(\mathrm{h}_0)\,\tilde{P}_4(\mathrm{h}_0 b)\,dV(\mathrm{h}_0). \quad (2)$$

The new elementary generalizations of the resolvent, which appear in Eq. (1) and (2), are defined in terms of infinite convergent series or solutions to systems of integral equations of two kinds: the equations of one kind are

$$\tilde{P}_1(c_0\mathrm{m}) = K_1(c_0\mathrm{m}) + \int_{F_1} \tilde{R}(c_1)\,K_1(c_0c_1)\,\tilde{P}_1(c_1\mathrm{m})\,dF(c_1) + \int_{V_1} \tilde{\beta}(\mathrm{h}_1)\,K_3(c_0\mathrm{h}_1)\,\tilde{P}_2(\mathrm{h}_1\mathrm{m})\,dV(\mathrm{h}_1); \quad (3)$$

$$\tilde{P}_2(\mathrm{h}_0\mathrm{m}) = K_2(\mathrm{h}_0\mathrm{m}) + \int_{F_1} \tilde{R}(c_1)\,K_2(\mathrm{h}_0c_1)\tilde{P}_1(c_1\mathrm{m})\,dF(c_1) + \int_{V_1} \tilde{\beta}(\mathrm{h}_1)\,K_4(\mathrm{h}_0\mathrm{h}_1)\,\tilde{P}_2(\mathrm{h}_1\mathrm{m})\,dV(\mathrm{h}_1); \quad (4)$$

$$\tilde{P}_3(c_0b) = K_3(c_0b) + \int_{F_1} \tilde{R}(c_1)\,K_1(c_0c_1)\,\tilde{P}_3(c_1b)\,dF(c_1) + \int_{V_1} \tilde{\beta}(\mathrm{h}_1)\,K_3(c_0\mathrm{h}_1)\,\tilde{P}_4(\mathrm{h}_1b)\,dV(\mathrm{h}_1); \quad (5)$$

$$\tilde{P}_4(\mathrm{h}_0b) = K_4(\mathrm{h}_0b) + \int_{F_1} \tilde{R}(c_1)\,K_2(\mathrm{h}_0c_1)\,\tilde{P}_3(c_1b)\,dF(c_1) + \int_{V_1} \tilde{\beta}(\mathrm{h}_1)\,K_4(\mathrm{h}_0\mathrm{h}_1)\,\tilde{P}_4(\mathrm{h}_1b)\,dV(\mathrm{h}_1). \quad (6)$$

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* A somewhat different equation analogous to (1) has been obtained by A. S. Nevskii [5, p. 71].

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The radiation-geometrical functions in these equations are defined by the following expressions:

$$K_1(c_0 m) = \cos\theta_{c_0} \cos\theta_m [\pi r^2_{c_0 m} \exp h_1]^{-1}, \qquad h_1 = \int_{r(c_0)}^{r(m)} k(r)\, dr; \tag{7}$$

$$K_2(h_0 m) = \cos\theta_m [\pi r^2_{h_0 m} \exp h_2]^{-1}, \qquad h_2 = \int_{r(h_0)}^{r(m)} k(r)\, dr; \tag{8}$$

$$K_3(c_0 b) = \cos\theta_{c_0} [\pi r^2_{c_0 b} \exp h_3]^{-1}, \qquad h_3 = \int_{r(c_0)}^{r(b)} k(r)\, dr; \tag{9}$$

$$K_4(h_0 b) = [\pi r^2_{h_0 b} \exp h_4]^{-1}, \qquad h_4 = \int_{r(h_0)}^{r(b)} k(r)\, dr. \tag{10}$$

We will divide the system into surface and volume zones so that the following conditions are satisfied at points within these zones:

a) radiation-geometrical conditions

$$K_1(F_i c_j) = K_1(F_i \bar{F}_j) = \frac{1}{F_j} K_1(F_i F_j) = \frac{1}{F_j} \int_{F_j} K_1(F_i c_j)\, dF(c_j), \tag{11}$$

$$K_2(V_z c_j) = K_2(V_z \bar{F}_j) = \frac{1}{F_j} K_2(V_z F_j) = \frac{1}{F_j} \int_{F_j} K_2(V_z c_j)\, dF(c_j), \tag{12}$$

$$K_3(F_i h_x) = K_3(F_i \bar{V}_x) = \frac{1}{V_x} K_3(F_i V_x) = \frac{1}{V_x} \int_{V_x} K_3(F_i h_x) dV(h_x), \tag{13}$$

$$K_4(V_z h_x) = K_4(V_z \bar{V}_x) = \frac{1}{V_x} K_4(V_z V_x) = \frac{1}{V_z} \int_{V_x} K_4(V_z h_x)\, dV(h_x), \tag{14}$$

where $K_1(F_i c_j)$ denotes the fraction of the flux radiating from surface zone F_i to a unit area of elementary surface $dF(c_j)$ with the center at point c_j, $K_1(F_i F_j)$ denotes the fraction of the flux radiating from surface zone F_i to surface zone F_j, and $K_1(F_i \bar{F}_j)$ denotes the fraction of the flux (mean) radiating from surface zone F_i to a unit area of surface zone F_j, the other functions in (11)-(14) having analogous physical meanings referred to volume zones only,

b) optical conditions

$$\tilde{R}(c_i) = \tilde{R}_i = \text{const}, \ i = 1, 2, \ldots, (m_1 + w_1); \qquad \tilde{\beta}(h_x) = \tilde{\beta}_x = \text{const}, \ x = 1, 2, \ldots, (m_2 + w_2); \tag{15}$$

c) energy conditions

$$\tilde{E}_c(c_i) = E_{c_i} = \text{const}, \quad \tilde{\eta}_c(h_x) = \tilde{\eta}_{c_x} = \text{const}. \tag{16}$$

For a more convenient further transformation of the integral equations defining the optics-geometrical resolvents, we assume that their local, integral, and mean values are defined according to the following expressions:

$$\tilde{P}_1(F_j c_i) = \int_{F_j} \tilde{P}_1(c_j c_i)\, dF(c_j), \quad \tilde{P}_2(V_z c_i) = \int_{V_z} \tilde{P}_2(h_z c_i)\, dV(h_z), \quad \tilde{P}_3(F_j h_x) = \int_{F_j} \tilde{P}_3(c_j h_x) dF(c_j), \quad \tilde{P}_4(V_z h_x) = \int_{V_z} \tilde{P}_4(h_z h_x)\, dV(h_z), \tag{17}$$

$$\tilde{P}_1(F_j F_i) = \int_{F_i} \tilde{P}_1(F_j c_i)\, dF(c_i), \quad \tilde{P}_2(V_z F_i) = \int_{F_i} \tilde{P}_2(V_z c_i)\, dF(c_i), \quad \tilde{P}_3(F_j V_x) = \int_{V_x} \tilde{P}_3(F_j h_x)\, dV(h_x), \quad \tilde{P}_4(V_z V_x) = \int_{V_x} \tilde{P}_4(V_z h_x) dV(h_x), \tag{18}$$

$$\tilde{P}_1(F_k \bar{F}_i) = \frac{1}{F_i} \tilde{P}_1(F_k F_i), \quad \tilde{P}_2(V_z \bar{F}_i) = \frac{1}{F_i} \tilde{P}_2(V_z F_i), \quad \tilde{P}_3(F_j \bar{V}_x) = \frac{1}{V_x} \tilde{P}_3(F_j V_x), \quad \tilde{P}_4(V_z \bar{V}_x) = \frac{1}{V_x} \tilde{P}_4(V_z V_x), \tag{19}$$

respectively.

From the system of integral equations (3)-(6), moreover, we obtain the following resolvent system of linear algebraic equations (20) and (21) for the new local and integral optics-geometrical resolvents respectively:

$$\begin{aligned}
\tilde{P}_1(F_k\mathrm{m}) &= K_1(F_k\mathrm{m}) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_1(F_k\overline{F}_j)\,\tilde{P}_1(F_j\mathrm{m}) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_3(F_k\overline{V}_y)\,\tilde{P}_2(V_y\mathrm{m});\\
\tilde{P}_2(V_z\mathrm{m}) &= K_2(V_z\mathrm{m}) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_2(V_z\overline{F}_j)\,\tilde{P}_1(F_j\mathrm{m}) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_4(V_z\overline{V}_y)\,\tilde{P}_2(V_y\mathrm{m});\\
\tilde{P}_3(F_kb) &= K_3(F_kb) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_1(F_k\overline{F}_j)\,\tilde{P}_3(F_jb) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_3(F_k\overline{V}_y)\,\tilde{P}_4(V_yb);\\
\tilde{P}_4(V_zb) &= K_4(V_zb) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_2(V_z\overline{F}_j)\,\tilde{P}_3(F_jb) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_4(V_z\overline{V}_y)\,\tilde{P}_4(V_yb);
\end{aligned} \tag{20}$$

$$\begin{aligned}
\tilde{P}_1(F_kF_i) &= K_1(F_kF_i) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_1(F_k\overline{F}_j)\,\tilde{P}_1(F_jF_i) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_3(F_k\overline{V}_y)\,\tilde{P}_2(V_yF_i);\\
\tilde{P}_2(V_zF_i) &= K_2(V_zF_i) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_2(V_z\overline{F}_j)\,\tilde{P}_1(F_jF_i) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_4(V_z\overline{V}_y)\,\tilde{P}_2(V_yF_i);\\
\tilde{P}_3(F_kV_x) &= K_2(F_kV_x) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_1(F_k\overline{F}_j)\tilde{P}_3(F_jV_x) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_3(F_k\overline{V}_y)\,\tilde{P}_4(V_yV_x);\\
\tilde{P}_4(V_zV_x) &= K_4(V_zV_x) + \sum_{j=1}^{m_1+w_1} \tilde{R}_j K_2(V_z\overline{F}_j)\tilde{P}(F_jV_x) + \sum_{y=1}^{m_2+w_2} \tilde{\beta}_y K_4(V_z\overline{V}_y)\,\tilde{P}_4(V_yV_x);
\end{aligned} \tag{21}$$

$$i,\ j,\ k = 1, 2, \ldots, (m_1 + w_1);\quad x, y, z = 1, 2, \ldots, (m_2 + w_2).$$

The equations which yield local, integral, and mean energy characteristics of heat radiation in such a system are obtained in the following form:

$$\begin{aligned}
E_{\mathrm{i}}(\mathrm{m}) &= \sum_{k=1}^{m_1+w_1} \tilde{E}_{ck}\,\tilde{P}_1(F_k\mathrm{m}) + \sum_{z=1}^{m_2+w_2} \eta_{cz}\,\tilde{P}_2(V_z\mathrm{m});\\
\eta_{\mathrm{i}}(b) &= \sum_{k=1}^{m_1+w_1} \tilde{E}_{ck}\,\tilde{P}_3(F_kb) + \sum_{z=1}^{m_2+w_2} \tilde{\eta}_{cz}\,\tilde{P}_4(V_zb);
\end{aligned} \tag{22}$$

$$\begin{aligned}
Q_{\mathrm{i}}(F_i) &= \int_{F_i} E_{\mathrm{i}}(c_i)\,dF(c_i) = \sum_{k=1}^{m_1+w_1} \tilde{E}_{ch}\,\tilde{P}_1(F_kF_i) + \sum_{z=1}^{m_2+w_2} \tilde{\eta}_{cz}\,\tilde{P}_2(V_zF_i);\\
Q_{\mathrm{i}}(V_x) &= \int_{V_x} \eta_{\mathrm{i}}(\mathrm{h}_x)\,dV(\mathrm{h}_x) = \sum_{k=1}^{m_1+w_1} \tilde{E}_{ch}\,\tilde{P}_1(F_kV_x) + \sum_{z=1}^{m_2+w_2} \tilde{\eta}_{cz}\,\tilde{P}_4(V_zV_x);
\end{aligned} \tag{23}$$

$$\begin{aligned}
&E_{\mathrm{i}}(\overline{F}_i) = \frac{1}{F_i} Q_{\mathrm{i}}(F_i);\quad \eta_{\mathrm{i}}(\overline{V}_x) = \frac{1}{V_x} Q_{\mathrm{i}}(V_x);\\
&i,\ j,\ k = 1, 2, \ldots, (m_1 + w_1);\quad x, y, z = 1, 2, \ldots, (m_2 + w_2).
\end{aligned} \tag{24}$$

respectively.

The following closure equations (25), (26), and (27) apply respectively to the local, the integral, and the mean new optics-geometrical resolvents:

$$\begin{aligned}
&\sum_{k=1}^{m_1} A_k\,\tilde{P}_1(F_k\mathrm{m}) + 4\sum_{z=1}^{m_2} \alpha_z\,\tilde{P}_2(V_z\mathrm{m}) = 1;\\
&\sum_{k=1}^{m_1} A_k\,P_3(F_kb) + 4\sum_{z=1}^{m_2} \alpha_z\,\tilde{P}_4(V_z\mathrm{m}) = 4;
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\sum_{k=1}^{m_1} A_k\,\tilde{P}_1(F_kF_i) + 4\sum_{z=1}^{m_2} \alpha_z\,P_2(V_zF_i) = F_i;\\
&\sum_{k=1}^{m_1} A_k\,P_1(F_kV_x) + 4\sum_{z=1}^{m_2} \alpha_z\,\tilde{P}_4(V_zV_x) = 4V_x;
\end{aligned} \tag{26}$$

$$\sum_{k=1}^{m_1} A_k \tilde{P}_1(F_k \bar{F}_i) + 4\sum_{z=1}^{m_2} \alpha_z \tilde{P}_2(V_z \bar{F}_i) = 1;$$
$$\sum_{k=1}^{m_1} A_k \tilde{P}_3(F_k \bar{V}_x) + 4\sum_{z=1}^{m_2} \alpha_z \tilde{P}_4(V_z \bar{V}_x) = 4. \quad (27)$$

The local generalized resolvents $\tilde{P}_1(F_i m)$ and $\tilde{P}_2(V_y m)$ denote the fractions of radiant energy which finally reach an elementary area dF(m) of the boundary surface from surface zone F_i and from volume zone V_y respectively, after an infinite number of reflections and absorptions at all boundaries and of dispersions and absorptions inside the entire medium of the system.

The results shown here define the conditions under which a system can be divided into separate zones, surface zones as well as volume zones, they also yield both local and mean characteristics of heat radiation at arbitrary boundary and inner points of a system which contains an absorbing and isotropically dispersing medium. The results may be used for determining the monochromatic energy characteristics of a system with selective optical properties of surfaces and media, or for determining their integral and mean values in systems with gray media.

NOTATION

$\eta_c(b)$	is the volume density of the intrinsic radiation at point b ($b \in V$);
V	is the volume;
F	is the surface;
$E_c(m)$	is the surface density of the intrinsic radiation at point m ($m \in F$);
dF(c)	is the elementary area on the boundary surface at point c ($c \in F$);
dV(h)	is the elementary volume at point h ($h \in V$);
θ_m	is the angle between the normal to the boundary surface at point m and the direction of the incident and departing radiation at that point;
r_{cm}	is the straight-line distance between points c and m;
r(c), r(h)	is the space coordinate along beam $\bar{r}_{cm}$ and $\bar{r}_{cb}$ or $\bar{r}_{hm}$ and $\bar{r}_{hb}$ at points c and h respectively;
h_i	is the optical length of beam path (i = 1, 2, 3, 4);
m_1	is the number of zones where densities of the intrinsic surface radiation are given and where the generalized reflection factor $\tilde{R}(c) = R(c) = 1 - A(c)$;
w_1	is the number of zones where densities of the resultant surface radiation are given and where the generalized reflection factor $\tilde{R}(c) = 1$;
m_2	is the number of zones where space densities of the intrinsic volume radiation are given and where the generalized dispersion factor for the medium $\tilde{\beta}(h) = \beta(h)$;
w_2	is the number of zones where space densities of the resultant volume radiation are given and where the generalized dispersion factor $\tilde{\beta}(h) = k(h) = \beta(h) + \alpha(h)$;
k(h)	is the decay factor;
$\alpha(h)$	is the absorption factor.

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